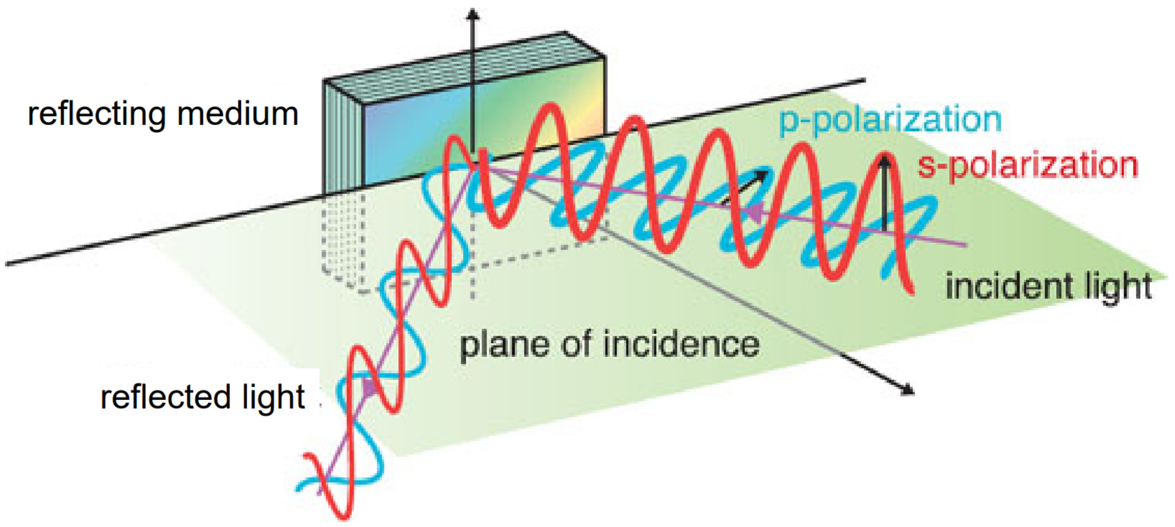
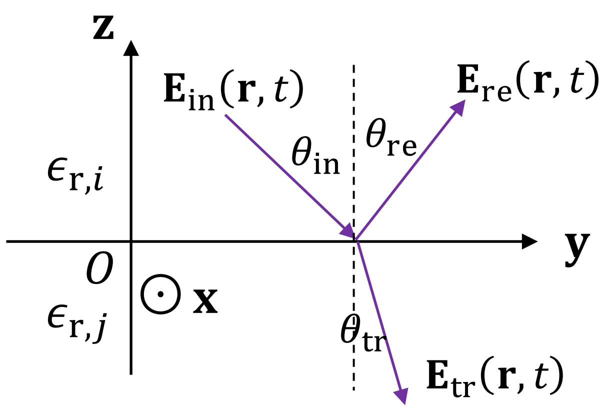
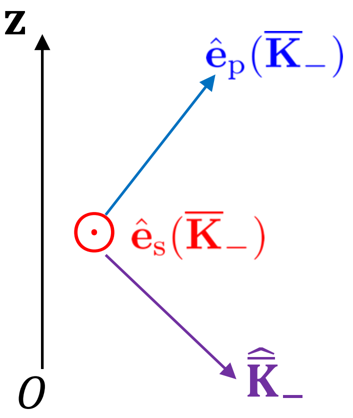
Definition of s-Polarized and p-Polarized Wave







Later we will study the propagation when space is divided into two, so we introduce the unit vectors of s-polarized and p-polarized electric fields.

$$\overline{\mathbf{K}}_{\pm} = k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}} \pm K_{z,i}(\omega)\hat{\mathbf{z}}$$

Similarly, for $z_R < 0$, we can also define the corresponding unit vectors of s- and p- electric fields.

$$\hat{\mathbf{e}}_{\mathrm{s}}(\overline{\mathbf{K}}_-) = \frac{\hat{\overline{\mathbf{K}}}_- \times \hat{\mathbf{z}}}{\left|\hat{\overline{\mathbf{K}}}_- \times \hat{\mathbf{z}}\right|} = \frac{k_y\hat{\mathbf{x}} - k_x\hat{\mathbf{y}}}{\sqrt{k_x^2 + k_y^2}} = \hat{\mathbf{e}}_{\mathrm{s}}(\overline{\mathbf{K}}_+)$$

$$\hat{\mathbf{e}}_{\mathrm{p}}(\overline{\mathbf{K}}_-) = \hat{\mathbf{e}}_{\mathrm{s}}(\overline{\mathbf{K}}_-) \times \hat{\overline{\mathbf{K}}}_- = \frac{K_{z,i}(\omega)\,(k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}})}{k_i(\omega)\sqrt{k_x^2 + k_y^2}} + \frac{\sqrt{k_x^2 + k_y^2}}{k_i(\omega)}\hat{\mathbf{z}}$$

$$\hat{\overline{\mathbf{K}}}_{\pm} \equiv \frac{\overline{\mathbf{K}}_{\pm}}{|\overline{\mathbf{K}}_{\pm}|} = \frac{\overline{\mathbf{K}}_{\pm}}{\sqrt{k_x^2 + k_y^2 + K_{z,i}^2(\omega)}} = \frac{\overline{\mathbf{K}}_{\pm}}{k_i(\omega)}$$

We can prove: $\bar{\bar{\mathbf{I}}} - \dfrac{\overline{\mathbf{K}}_-\overline{\mathbf{K}}_-}{k_i^2(\omega)} = \bar{\bar{\mathbf{I}}} - \hat{\overline{\mathbf{K}}}_-\hat{\overline{\mathbf{K}}}_- = \hat{\mathbf{e}}_{\mathrm{s}}(\overline{\mathbf{K}}_-)\hat{\mathbf{e}}_{\mathrm{s}}(\overline{\mathbf{K}}_-) + \hat{\mathbf{e}}_{\mathrm{p}}(\overline{\mathbf{K}}_-)\hat{\mathbf{e}}_{\mathrm{p}}(\overline{\mathbf{K}}_-)$

Physical Picture of the Spectral Representation

Similarly, for $z_R > 0$, we can also define the corresponding unit vectors of s- and p- electric fields.

$$\hat{\mathbf{e}}_\mathrm{s}(\overline{\mathbf{K}}_+) \equiv \frac{\hat{\overline{\mathbf{K}}}_+ \times \hat{\mathbf{z}}}{\left|\hat{\overline{\mathbf{K}}}_+ \times \hat{\mathbf{z}}\right|} = \frac{k_y \hat{\mathbf{x}} - k_x \hat{\mathbf{y}}}{\sqrt{k_x^2 + k_y^2}} \qquad \hat{\mathbf{e}}_\mathrm{p}(\overline{\mathbf{K}}_+) \equiv \hat{\mathbf{e}}_\mathrm{s}(\overline{\mathbf{K}}_+) \times \hat{\overline{\mathbf{K}}}_+ = \frac{-K_{z,i}(\omega)\,(k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}})}{k_i(\omega)\sqrt{k_x^2 + k_y^2}} + \frac{\sqrt{k_x^2 + k_y^2}}{k_i(\omega)}\hat{\mathbf{z}}$$

We can also prove: $\overline{\overline{\mathbf{I}}} - \dfrac{\overline{\mathbf{K}}_+ \overline{\mathbf{K}}_+}{k_i^2(\omega)} = \overline{\overline{\mathbf{I}}} - \hat{\overline{\mathbf{K}}}_+ \hat{\overline{\mathbf{K}}}_+ = \hat{\mathbf{e}}_\mathrm{s}(\overline{\mathbf{K}}_+)\hat{\mathbf{e}}_\mathrm{s}(\overline{\mathbf{K}}_+) + \hat{\mathbf{e}}_\mathrm{p}(\overline{\mathbf{K}}_+)\hat{\mathbf{e}}_\mathrm{p}(\overline{\mathbf{K}}_+)$

Therefore:

$$\overline{\overline{\mathbf{G}}}_0^{(i)}(\mathbf{r}, \mathbf{r}', \omega) = \frac{-\hat{\mathbf{z}}\hat{\mathbf{z}}}{k_i^2(\omega)}\delta(\mathbf{R}) + \begin{cases} \dfrac{i}{8\pi^2} \displaystyle\iint_{-\infty}^{+\infty} \dfrac{dk_x dk_y}{K_{z,i}(\omega)} \left[\hat{\mathbf{e}}_\mathrm{s}(\overline{\mathbf{K}}_+)\hat{\mathbf{e}}_\mathrm{s}(\overline{\mathbf{K}}_+) + \hat{\mathbf{e}}_\mathrm{p}(\overline{\mathbf{K}}_+)\hat{\mathbf{e}}_\mathrm{p}(\overline{\mathbf{K}}_+)\right] e^{i\overline{\mathbf{K}}_+ \cdot \mathbf{R}} & z_R > 0 \\ \dfrac{i}{8\pi^2} \displaystyle\iint_{-\infty}^{+\infty} \dfrac{dk_x dk_y}{K_{z,i}(\omega)} \left[\hat{\mathbf{e}}_\mathrm{s}(\overline{\mathbf{K}}_-)\hat{\mathbf{e}}_\mathrm{s}(\overline{\mathbf{K}}_-) + \hat{\mathbf{e}}_\mathrm{p}(\overline{\mathbf{K}}_-)\hat{\mathbf{e}}_\mathrm{p}(\overline{\mathbf{K}}_-)\right] e^{i\overline{\mathbf{K}}_- \cdot \mathbf{R}} & z_R < 0 \end{cases}$$

[Principles of Nano-Optics by Lukas Novotny Eq.(10.15)]

where: $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ $\quad \overline{\mathbf{K}}_\pm = k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}} \pm K_{z,i}(\omega)\hat{\mathbf{z}}$

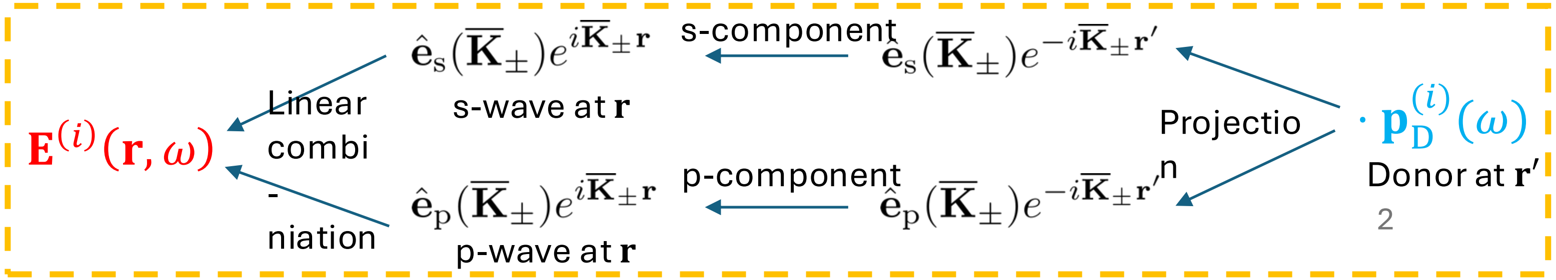
$\mathbf{r} = (x, y, z) \quad \mathbf{r}' = (x', y', z')$

Alternatively ($\mathbf{R} \neq \mathbf{0}$):

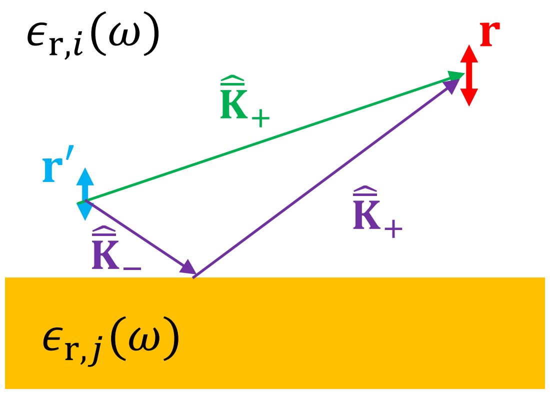
$$\overline{\overline{\mathbf{G}}}_0^{(i)}(\mathbf{r}, \mathbf{r}', \omega) = \begin{cases} \dfrac{i}{8\pi^2} \displaystyle\iint_{-\infty}^{+\infty} \dfrac{dk_x dk_y}{K_{z,i}(\omega)} \left\{\left[\hat{\mathbf{e}}_\mathrm{s}(\overline{\mathbf{K}}_+)e^{i\overline{\mathbf{K}}_+\cdot\mathbf{r}}\right]\left[\hat{\mathbf{e}}_\mathrm{s}(\overline{\mathbf{K}}_+)e^{-i\overline{\mathbf{K}}_+\cdot\mathbf{r}'}\right] + \left[\hat{\mathbf{e}}_\mathrm{p}(\overline{\mathbf{K}}_+)e^{i\overline{\mathbf{K}}_+\cdot\mathbf{r}}\right]\left[\hat{\mathbf{e}}_\mathrm{p}(\overline{\mathbf{K}}_+)e^{-i\overline{\mathbf{K}}_+\cdot\mathbf{r}'}\right]\right\} & z > z' \\ \dfrac{i}{8\pi^2} \displaystyle\iint_{-\infty}^{+\infty} \dfrac{dk_x dk_y}{K_{z,i}(\omega)} \left\{\left[\hat{\mathbf{e}}_\mathrm{s}(\overline{\mathbf{K}}_-)e^{i\overline{\mathbf{K}}_-\cdot\mathbf{r}}\right]\left[\hat{\mathbf{e}}_\mathrm{s}(\overline{\mathbf{K}}_-)e^{-i\overline{\mathbf{K}}_-\cdot\mathbf{r}'}\right] + \left[\hat{\mathbf{e}}_\mathrm{p}(\overline{\mathbf{K}}_-)e^{i\overline{\mathbf{K}}_-\cdot\mathbf{r}}\right]\left[\hat{\mathbf{e}}_\mathrm{p}(\overline{\mathbf{K}}_-)e^{-i\overline{\mathbf{K}}_-\cdot\mathbf{r}'}\right]\right\} & z < z' \end{cases}$$

Recall:

$$\mathbf{E}^{(i)}(\mathbf{r}, \omega) = \frac{\omega^2}{\epsilon_0 c^2}\overline{\overline{\mathbf{G}}}^{(i)}(\mathbf{r}, \mathbf{r}', \omega) \cdot \mathbf{p}_\mathrm{D}^{(i)}(\omega)$$



Dyadic Green's Function for Two Layer Medium [Spectral Representation]



Electric field propagates along purple path:

$$\overline{\overline{\mathbf{G}}}^{(i)}_{\text{refl}}(\mathbf{r},\mathbf{r}',\omega) = \frac{i}{8\pi^2}\iint_{-\infty}^{+\infty}\frac{dk_x dk_y}{K_{z,i}(\omega)}\left\{\left[R_\text{s}\hat{\mathbf{e}}_\text{s}(\overline{\mathbf{K}}_+)e^{i\overline{\mathbf{K}}_+\cdot\mathbf{r}}\right]\left[\hat{\mathbf{e}}_\text{s}(\overline{\mathbf{K}}_-)e^{-i\overline{\mathbf{K}}_-\cdot\mathbf{r}'}\right] + \left[R_\text{p}\hat{\mathbf{e}}_\text{p}(\overline{\mathbf{K}}_+)e^{i\overline{\mathbf{K}}_+\cdot\mathbf{r}}\right]\left[\hat{\mathbf{e}}_\text{p}(\overline{\mathbf{K}}_-)e^{-i\overline{\mathbf{K}}_-\cdot\mathbf{r}'}\right]\right\}$$

$$= \frac{i}{8\pi^2}\iint_{-\infty}^{+\infty} dk_x dk_y \left\{\frac{R_\text{s}}{K_{z,i}(\omega)k_\rho^2}\begin{bmatrix} k_y^2 & -k_x k_y & 0 \\ -k_x k_y & k_x^2 & 0 \\ 0 & 0 & 0\end{bmatrix} + \frac{-R_\text{p}}{k_i^2(\omega)k_\rho^2}\begin{bmatrix} k_x^2 K_{z,i}(\omega) & k_x k_y K_{z,i}(\omega) & k_x k_\rho^2 \\ k_x k_y K_{z,i}(\omega) & k_y^2 K_{z,i}(\omega) & k_y k_\rho^2 \\ -k_x k_\rho^2 & -k_y k_\rho^2 & -k_\rho^4/K_{z,i}(\omega)\end{bmatrix}\right\} e^{i\overline{\mathbf{K}}_+\cdot\mathbf{r}}e^{-i\overline{\mathbf{K}}_-\cdot\mathbf{r}'}$$

$$= \frac{i}{8\pi^2}\iint_{-\infty}^{+\infty} dk_x dk_y \left\{\overline{\overline{\mathbf{M}}}_\text{s}(k_x,k_y,\omega) + \overline{\overline{\mathbf{M}}}_\text{p}(k_x,k_y,\omega)\right\} e^{ik_x(x-x')+ik_y(y-y')}e^{iK_{z,i}(\omega)(z+z')}$$

where: $k_\rho^2 = k_x^2 + k_y^2$

We can prove that $\overline{\overline{\mathbf{G}}}^{(i)}_{\text{refl}}(\mathbf{r},\mathbf{r}',\omega)$ has the same form for $z < z'$. (see Lecture note by K. Sarabandi)

Electric field propagates along green path:

$$\overline{\overline{\mathbf{G}}}^{(i)}_0(\mathbf{r},\mathbf{r}',\omega) = \frac{i}{8\pi^2}\iint_{-\infty}^{+\infty}\frac{dk_x dk_y}{K_{z,i}(\omega)}\left\{\left[\hat{\mathbf{e}}_\text{s}(\overline{\mathbf{K}}_+)e^{i\overline{\mathbf{K}}_+\cdot\mathbf{r}}\right]\left[\hat{\mathbf{e}}_\text{s}(\overline{\mathbf{K}}_+)e^{-i\overline{\mathbf{K}}_+\cdot\mathbf{r}'}\right] + \left[\hat{\mathbf{e}}_\text{p}(\overline{\mathbf{K}}_+)e^{i\overline{\mathbf{K}}_+\cdot\mathbf{r}}\right]\left[\hat{\mathbf{e}}_\text{p}(\overline{\mathbf{K}}_+)e^{-i\overline{\mathbf{K}}_+\cdot\mathbf{r}'}\right]\right\}$$

Therefore:

$$\overline{\overline{\mathbf{G}}}^{(i)}(\mathbf{r},\mathbf{r}',\omega) = \overline{\overline{\mathbf{G}}}^{(i)}_0(\mathbf{r},\mathbf{r}',\omega) + \overline{\overline{\mathbf{G}}}^{(i)}_{\text{refl}}(\mathbf{r},\mathbf{r}',\omega)$$

Boundary Condition

Maxwell's equations in **time domain** (curl part):

(a)
$$\nabla \times \mathbf{E}(\mathbf{r},t) = -\partial_t \mathbf{B}(\mathbf{r},t)$$
$$\nabla \times \mathbf{H}(\mathbf{r},t) = \partial_t \mathbf{D}(\mathbf{r},t)$$

We apply Stokes' theorem on the left-hand side of equation:

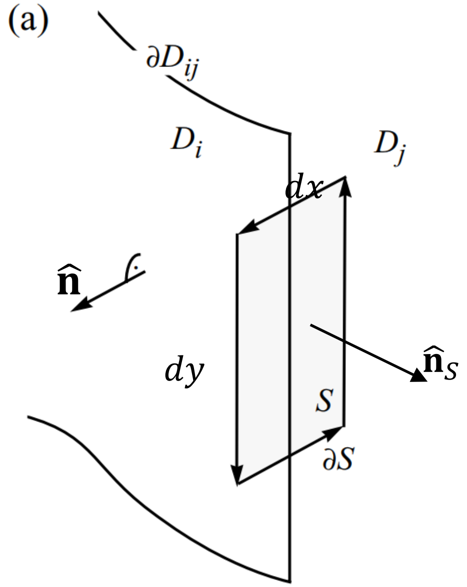
$$\oint_{\partial S} \mathbf{E}(\mathbf{r},t) \cdot d\mathbf{s} = -\iint_S \frac{\partial}{\partial t} \mathbf{B}(\mathbf{r},t) \cdot \hat{\mathbf{n}}_S da$$
$$\oint_{\partial S} \mathbf{H}(\mathbf{r},t) \cdot d\mathbf{s} = \iint_S \frac{\partial}{\partial t} \mathbf{D}(\mathbf{r},t) \cdot \hat{\mathbf{n}}_S da$$

When $S \to 0$:

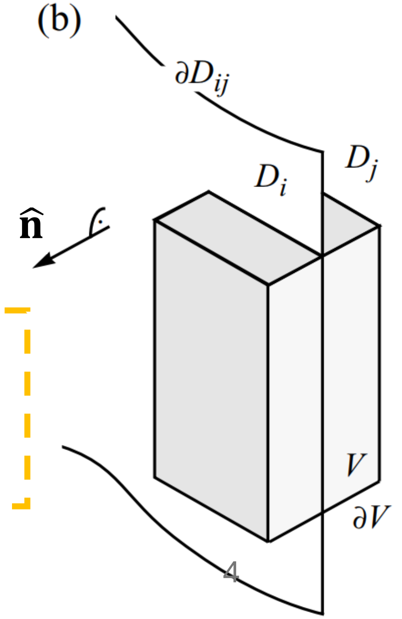
$$\hat{\mathbf{n}} \times \left(\mathbf{E}^{(i)}(\mathbf{r},t) - \mathbf{E}^{(j)}(\mathbf{r},t)\right) = 0$$
$$\hat{\mathbf{n}} \times \left(\mathbf{H}^{(i)}(\mathbf{r},t) - \mathbf{H}^{(j)}(\mathbf{r},t)\right) = 0$$

where: $\mathbf{r} \in \partial D_{ij}$

[Principles of Nano-Optics by Lukas Novotny Eq.(2.39)-(2.42)]



Integration paths for the derivation of the boundary conditions on the interface ∂D_{ij} between two adjacent domains D_i and D_j.



Maxwell's equations in **time domain** (divergence part):

(b)
$$\nabla \cdot \mathbf{D}(\mathbf{r},t) = 0$$
$$\nabla \cdot \mathbf{B}(\mathbf{r},t) = 0$$

We apply Gauss's theorem on the left-hand side of equation:

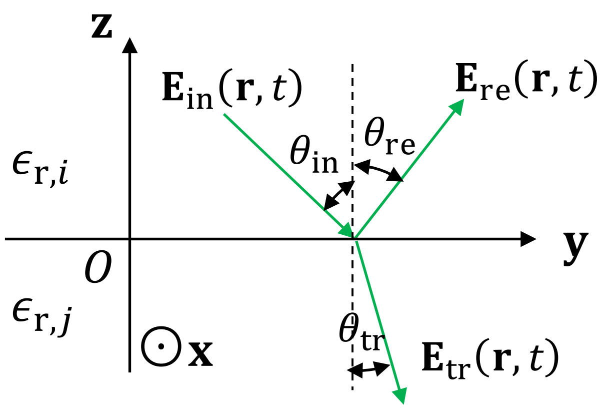
$$\iint_{\partial V} \mathbf{D}(\mathbf{r},t) = 0$$
$$\iint_{\partial V} \mathbf{B}(\mathbf{r},t) = 0$$

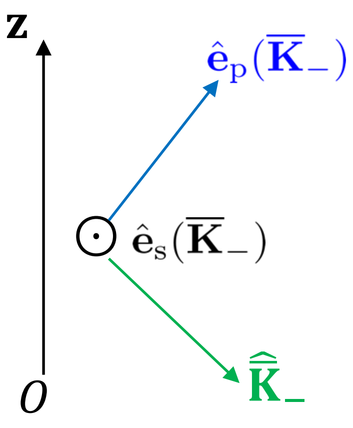
When $V \to 0$:

$$\hat{\mathbf{n}} \cdot \left(\mathbf{D}^{(i)}(\mathbf{r},t) - \mathbf{D}^{(j)}(\mathbf{r},t)\right) = 0$$
$$\hat{\mathbf{n}} \cdot \left(\mathbf{B}^{(i)}(\mathbf{r},t) - \mathbf{B}^{(j)}(\mathbf{r},t)\right) = 0$$

where: $\mathbf{r} \in \partial D_{ij}$

Law of Reflection and Refraction:





Without loss of generality, the incident, reflected, and transmitted electric field are monochromic plane waves.

$$\mathbf{E}_{\rm in}(\mathbf{r},t) = \mathbf{E}_{\rm in} e^{i(\mathbf{k}_{\rm in}\cdot\mathbf{r} - \omega_{\rm in} t)} \qquad \mathbf{E}_{\rm re}(\mathbf{r},t) = \mathbf{E}_{\rm re} e^{i(\mathbf{k}_{\rm re}\cdot\mathbf{r} - \omega_{\rm re} t)} \qquad \mathbf{E}_{\rm tr}(\mathbf{r},t) = \mathbf{E}_{\rm tr} e^{i(\mathbf{k}_{\rm tr}\cdot\mathbf{r} - \omega_{\rm tr} t)}$$

The boundary condition of Maxwell's equations give: $\hat{\mathbf{z}} \times [\mathbf{E}_{\rm in}(\mathbf{r},t) + \mathbf{E}_{\rm re}(\mathbf{r},t)] = \hat{\mathbf{z}} \times \mathbf{E}_{\rm tr}(\mathbf{r},t)$ (for all time t and all boundary points $\mathbf{r}$)

$$\hat{\mathbf{z}} \times \mathbf{E}_{\rm in} e^{i(\mathbf{k}_{\rm in}\cdot\mathbf{r} - \omega_{\rm in} t)} + \hat{\mathbf{z}} \times \mathbf{E}_{\rm re} e^{i(\mathbf{k}_{\rm re}\cdot\mathbf{r} - \omega_{\rm re} t)} = \hat{\mathbf{z}} \times \mathbf{E}_{\rm tr} e^{i(\mathbf{k}_{\rm tr}\cdot\mathbf{r} - \omega_{\rm tr} t)}$$

Implies the incident, reflected, and transmitted wave have the same polarization direction.

Then the phase matching gives $i(\mathbf{k}_{\rm in}\cdot\mathbf{r} - \omega_{\rm in} t) = i(\mathbf{k}_{\rm re}\cdot\mathbf{r} - \omega_{\rm re} t) = i(\mathbf{k}_{\rm tr}\cdot\mathbf{r} - \omega_{\rm tr} t)$

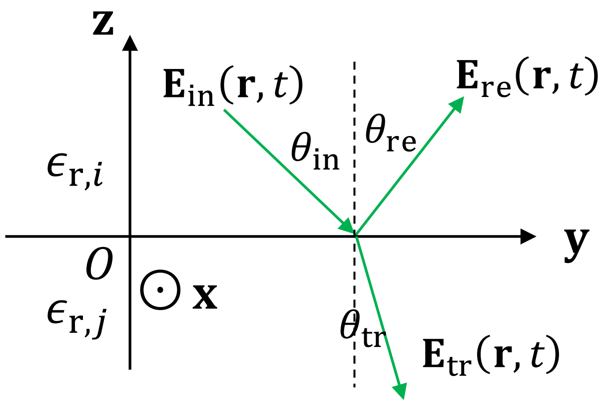
At $\mathbf{r} = 0$: $\quad -i\omega_{\rm in} t = -i\omega_{\rm re} t = -i\omega_{\rm tr} t$ (for all time t) $\quad \therefore \quad \omega_{\rm in} = \omega_{\rm re} = \omega_{\rm tr} \equiv \omega_0$

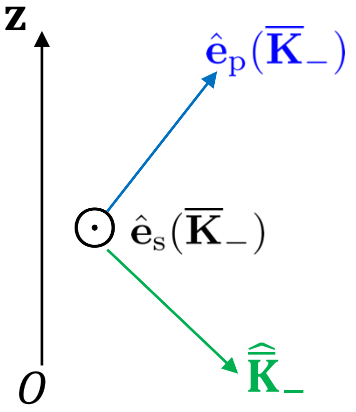
At $t = 0$: $\quad i\mathbf{k}_{\rm in}\cdot\mathbf{r} = i\mathbf{k}_{\rm re}\cdot\mathbf{r} = i\mathbf{k}_{\rm tr}\cdot\mathbf{r}$ (for all boundary points $\mathbf{r}$) $\quad \therefore \quad \mathbf{k}_{\rm in}\cdot\mathbf{r} = \mathbf{k}_{\rm re}\cdot\mathbf{r} = \mathbf{k}_{\rm tr}\cdot\mathbf{r}$ $\quad$ where: $|\mathbf{k}_{\rm in}| = \frac{\sqrt{\epsilon_{{\rm r},i}}\,\omega_0}{c}$

Law of reflection: $\quad |\mathbf{k}_{\rm in}||\mathbf{r}|\cos\left(\frac{\pi}{2} - \theta_{\rm in}\right) = |\mathbf{k}_{\rm re}||\mathbf{r}|\cos\left(\frac{\pi}{2} - \theta_{\rm re}\right) \quad \therefore \quad \sin(\theta_{\rm in}) = \sin(\theta_{\rm re})$ $\quad$ or $\quad \theta_{\rm in} = \theta_{\rm re}$

Law of refraction: $\quad |\mathbf{k}_{\rm in}||\mathbf{r}|\cos\left(\frac{\pi}{2} - \theta_{\rm in}\right) = |\mathbf{k}_{\rm tr}||\mathbf{r}|\cos\left(\frac{\pi}{2} - \theta_{\rm tr}\right) \quad \therefore \quad \sqrt{\epsilon_{{\rm r},i}}\sin(\theta_{\rm in}) = \sqrt{\epsilon_{{\rm r},j}}\sin(\theta_{\rm tr})$ $\quad$ or $\quad n_i \sin(\theta_{\rm in}) = n_j \sin(\theta_{\rm tr})$

Fresnel Equations of s-Wave





$$\mathbf{E}_{in}(\mathbf{r},t) = \mathbf{E}_{in} e^{i(\mathbf{k}_{in}\cdot\mathbf{r}-\omega_0 t)}$$
$$\mathbf{E}_{re}(\mathbf{r},t) = \mathbf{E}_{re} e^{i(\mathbf{k}_{re}\cdot\mathbf{r}-\omega_0 t)}$$
$$\mathbf{E}_{tr}(\mathbf{r},t) = \mathbf{E}_{tr} e^{i(\mathbf{k}_{tr}\cdot\mathbf{r}-\omega_0 t)}$$

$$F(\omega) \equiv \int_{-\infty}^{\infty} f(t) e^{i\omega t}\, \mathrm{d}t$$

$$\mathbf{E}_{in}(\mathbf{r},\omega) = \mathbf{E}_{in} e^{i\mathbf{k}_{in}\cdot\mathbf{r}} \delta(\omega-\omega_0)$$
$$\mathbf{E}_{re}(\mathbf{r},\omega) = \mathbf{E}_{re} e^{i\mathbf{k}_{re}\cdot\mathbf{r}} \delta(\omega-\omega_0)$$
$$\mathbf{E}_{tr}(\mathbf{r},\omega) = \mathbf{E}_{tr} e^{i\mathbf{k}_{tr}\cdot\mathbf{r}} \delta(\omega-\omega_0)$$

For s-wave, $\mathbf{E}^s_{in}(\mathbf{r},\omega) = E^s_{in}\hat{\boldsymbol{x}} e^{i\mathbf{k}_{in}\cdot\mathbf{r}}\delta(\omega-\omega_0)$ $\quad \mathbf{E}^s_{re}(\mathbf{r},\omega) = E^s_{re}\hat{\boldsymbol{x}} e^{i\mathbf{k}_{re}\cdot\mathbf{r}}\delta(\omega-\omega_0)$ $\quad \mathbf{E}^s_{tr}(\mathbf{r},\omega) = E^s_{tr}\hat{\boldsymbol{x}} e^{i\mathbf{k}_{tr}\cdot\mathbf{r}}\delta(\omega-\omega_0)$

$\because$ Boundary condition: $\hat{\mathbf{z}} \times \left(\mathbf{E}^{(i)}(\mathbf{r},t) - \mathbf{E}^{(j)}(\mathbf{r},t)\right) = 0$ $\quad \therefore$ $\hat{\boldsymbol{y}}\left(E^s_{in} e^{i\mathbf{k}_{in}\cdot\mathbf{r}} + E^s_{re} e^{i\mathbf{k}_{re}\cdot\mathbf{r}} - E^s_{tr} e^{i\mathbf{k}_{tr}\cdot\mathbf{r}}\right)\delta(\omega-\omega_0) = 0$

$$\therefore \quad E^s_{in} + E^s_{re} = E^s_{tr}$$

$\because$ $\nabla \times \mathbf{E}(\mathbf{r},\omega) = i\omega \mathbf{B}(\mathbf{r},\omega)$

$\mathbf{H}(\mathbf{r},\omega) = \mu_0^{-1}\mathbf{B}(\mathbf{r},\omega)$

$\therefore$

$$\mathbf{H}^s_{in}(\mathbf{r},\omega) = E^s_{in}(k_{in,z}\hat{\boldsymbol{y}} - k_{in,y}\hat{\mathbf{z}}) e^{i\mathbf{k}_{in}\cdot\mathbf{r}}\delta(\omega-\omega_0)/\omega\mu_0$$
$$\mathbf{H}^s_{re}(\mathbf{r},\omega) = E^s_{re}(k_{re,z}\hat{\boldsymbol{y}} - k_{re,y}\hat{\mathbf{z}}) e^{i\mathbf{k}_{re}\cdot\mathbf{r}}\delta(\omega-\omega_0)/\omega\mu_0$$
$$\mathbf{H}^s_{tr}(\mathbf{r},\omega) = E^s_{tr}(k_{tr,z}\hat{\boldsymbol{y}} - k_{tr,y}\hat{\mathbf{z}}) e^{i\mathbf{k}_{tr}\cdot\mathbf{r}}\delta(\omega-\omega_0)/\omega\mu_0$$

$$\therefore \quad k_{in,z}E^s_{in} + k_{re,z}E^s_{re} = k_{tr,z}E^s_{tr}$$

$\because$ Boundary condition: $\hat{\mathbf{z}} \times \left(\mathbf{H}^{(i)}(\mathbf{r},t) - \mathbf{H}^{(j)}(\mathbf{r},t)\right) = 0$ $\quad \therefore$ $\dfrac{\hat{\boldsymbol{x}}\left(E^s_{in}k_{in,z} e^{i\mathbf{k}_{in}\cdot\mathbf{r}} + E^s_{re}k_{re,z} e^{i\mathbf{k}_{re}\cdot\mathbf{r}} - E^s_{tr}k_{tr,z} e^{i\mathbf{k}_{tr}\cdot\mathbf{r}}\right)\delta(\omega-\omega_0)}{-\omega\mu_0} = 0$

$\because$

$$k_{in,z} = -k_{re,z} = \sqrt{\epsilon_{r,i}(\omega)\omega^2/c^2 - k_x^2 - k_y^2} \equiv -K_{z,i}(\omega)$$
$$k_{tr,z} = \sqrt{\epsilon_{r,j}(\omega)\omega^2/c^2 - k_x^2 - k_y^2} \equiv -K_{z,j}(\omega)$$

Combining the results in two orange boxes:

$$K_{z,i}(\omega)(E^s_{in} - E^s_{re}) = K_{z,j}(\omega)(E^s_{in} + E^s_{re})$$

$$\therefore R_s = \frac{E^s_{re}}{E^s_{in}} = \frac{K_{z,i}(\omega) - K_{z,j}(\omega)}{K_{z,i}(\omega) + K_{z,j}(\omega)}$$

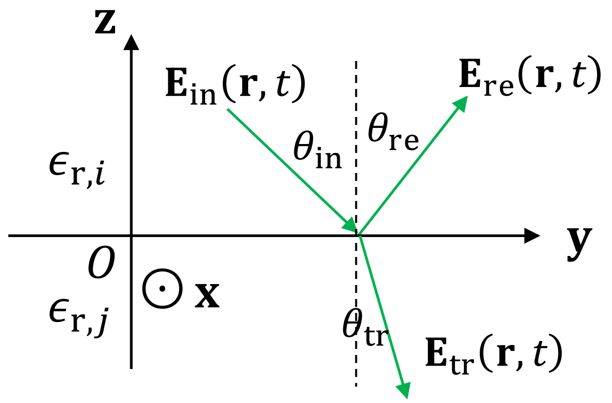
Fresnel Equations of p-Wave

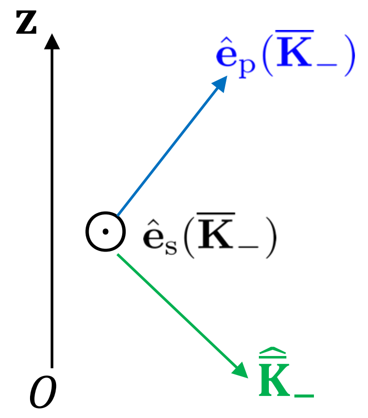
$$\hat{\mathbf{e}}_{\mathrm{p}}(\overline{\mathbf{K}}^{i}_{\pm}) = \mp\frac{K_{z,i}(\omega)\,(k_y\hat{\mathbf{y}})}{k_i(\omega)\sqrt{k_y^2}} + \frac{\sqrt{k_y^2}}{k_i(\omega)}\hat{\mathbf{z}} = \mp\frac{K_{z,i}(\omega)}{k_i(\omega)}\hat{\mathbf{y}} + \frac{k_y}{k_i(\omega)}\hat{\mathbf{z}}$$

where:

$$k_i(\omega) = \sqrt{\epsilon_{\mathrm{r,i}}(\omega)}\frac{\omega}{c}$$

$$\overline{\mathbf{K}}^{i}_{\pm} = k_y\hat{\mathbf{y}} \pm K_{z,i}(\omega)\hat{\mathbf{z}}$$





For p-wave, $\mathbf{E}^{\mathrm{p}}_{\mathrm{in}}(\mathbf{r},\omega) = E^{\mathrm{p}}_{\mathrm{in}}\hat{\boldsymbol{e}}_{\mathrm{p}}(\overline{\mathbf{K}}^{i}_{-})e^{i\overline{\mathbf{K}}^{i}_{-}\cdot\mathbf{r}}\delta(\omega-\omega_0)$ $\quad\mathbf{E}^{\mathrm{p}}_{\mathrm{re}}(\mathbf{r},\omega) = E^{\mathrm{p}}_{\mathrm{re}}\hat{\boldsymbol{e}}_{\mathrm{p}}(\overline{\mathbf{K}}^{i}_{+})e^{i\overline{\mathbf{K}}^{i}_{+}\cdot\mathbf{r}}\delta(\omega-\omega_0)$ $\quad\mathbf{E}^{\mathrm{p}}_{\mathrm{tr}}(\mathbf{r},\omega) = E^{\mathrm{p}}_{\mathrm{tr}}\hat{\boldsymbol{e}}_{\mathrm{p}}(\overline{\mathbf{K}}^{j}_{-})e^{i\overline{\mathbf{K}}^{j}_{-}\cdot\mathbf{r}}\delta(\omega-\omega_0)$

∵ Boundary condition: $\hat{\mathbf{z}}\times\left(\mathbf{E}^{(i)}(\mathbf{r},t) - \mathbf{E}^{(j)}(\mathbf{r},t)\right) = 0$

$$\therefore \quad E^{\mathrm{p}}_{\mathrm{in}}\frac{K_{z,i}(\omega)}{k_i(\omega)} + E^{\mathrm{p}}_{\mathrm{re}}\frac{-K_{z,i}(\omega)}{k_i(\omega)} = E^{\mathrm{p}}_{\mathrm{tr}}\frac{K_{z,j}(\omega)}{k_j(\omega)}$$

∵ $\nabla\times\mathbf{E}(\mathbf{r},\omega) = i\omega\mathbf{B}(\mathbf{r},\omega)$

$\mathbf{H}(\mathbf{r},\omega) = \mu_0^{-1}\mathbf{B}(\mathbf{r},\omega)$

$$i\omega\mu_0\mathbf{H}^{\mathrm{p}}_{\mathrm{in}}(\mathbf{r},\omega) = i\hat{\boldsymbol{x}}E^{\mathrm{p}}_{\mathrm{in}}\frac{k_y^2+K_{z,i}^2(\omega)}{k_i(\omega)}e^{i\overline{\mathbf{K}}^{i}_{-}\cdot\mathbf{r}}\delta(\omega-\omega_0) = i\hat{\boldsymbol{x}}E^{\mathrm{p}}_{\mathrm{in}}\frac{k_i^2(\omega)}{k_i(\omega)}e^{i\overline{\mathbf{K}}^{i}_{-}\cdot\mathbf{r}}\delta(\omega-\omega_0)$$

$$i\omega\mu_0\mathbf{H}^{\mathrm{p}}_{\mathrm{re}}(\mathbf{r},\omega) = i\hat{\boldsymbol{x}}E^{\mathrm{p}}_{\mathrm{re}}\frac{k_y^2+K_{z,i}^2(\omega)}{k_i(\omega)}e^{i\overline{\mathbf{K}}^{i}_{+}\cdot\mathbf{r}}\delta(\omega-\omega_0) = i\hat{\boldsymbol{x}}E^{\mathrm{p}}_{\mathrm{re}}\frac{k_i^2(\omega)}{k_i(\omega)}e^{i\overline{\mathbf{K}}^{i}_{+}\cdot\mathbf{r}}\delta(\omega-\omega_0)$$

$$i\omega\mu_0\mathbf{H}^{\mathrm{p}}_{\mathrm{tr}}(\mathbf{r},\omega) = i\hat{\boldsymbol{x}}E^{\mathrm{p}}_{\mathrm{tr}}\frac{k_j^2(\omega)}{k_j(\omega)}e^{i\overline{\mathbf{K}}^{j}_{-}\cdot\mathbf{r}}\delta(\omega-\omega_0)$$

∵ Boundary condition:

$$\hat{\mathbf{z}}\times\left(\mathbf{H}^{(i)}(\mathbf{r},t) - \mathbf{H}^{(j)}(\mathbf{r},t)\right) = 0$$

$$\therefore\ E^{\mathrm{p}}_{\mathrm{in}}k_i(\omega) + E^{\mathrm{p}}_{\mathrm{re}}k_i(\omega) = E^{\mathrm{p}}_{\mathrm{tr}}k_j(\omega)$$

Combining the results in two orange boxes:

$$\therefore\quad R_{\mathrm{p}} = \frac{E^{\mathrm{p}}_{\mathrm{re}}}{E^{\mathrm{p}}_{\mathrm{in}}} = \frac{k_j^2(\omega)K_{z,i}(\omega) - k_i^2(\omega)K_{z,j}(\omega)}{k_j^2(\omega)K_{z,i}(\omega) + k_i^2(\omega)K_{z,j}(\omega)} = \frac{\epsilon_{\mathrm{r},j}(\omega)K_{z,i}(\omega) - \epsilon_{\mathrm{r},i}(\omega)K_{z,j}(\omega)}{\epsilon_{\mathrm{r},j}(\omega)K_{z,i}(\omega) + \epsilon_{\mathrm{r},i}(\omega)K_{z,j}(\omega)}$$

The Dyadic Green's Function in Cylindrical Coordinates

$$\overline{\overline{\mathbf{G}}}^{(i)}_{\text{refl}}(\mathbf{r},\mathbf{r}',\omega) = \frac{i}{8\pi^2}\iint_{-\infty}^{+\infty} dk_x dk_y \left\{\overline{\overline{\mathbf{M}}}_{\text{s}}(k_x,k_y,\omega)+\overline{\overline{\mathbf{M}}}_{\text{p}}(k_x,k_y,\omega)\right\} e^{ik_x(x-x')+ik_y(y-y')}e^{iK_{z,i}(\omega)(z+z')}$$

where:

$$\overline{\overline{\mathbf{M}}}_{\text{s}}(k_x,k_y,\omega) = \frac{R_{\text{s}}}{K_{z,i}(\omega)k_\rho^2}\begin{bmatrix} k_y^2 & -k_x k_y & 0 \\ -k_x k_y & k_x^2 & 0 \\ 0 & 0 & 0\end{bmatrix}$$

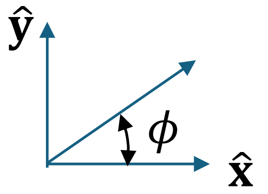
$$\overline{\overline{\mathbf{M}}}_{\text{p}}(k_x,k_y,\omega) = \frac{-R_{\text{p}}}{k_i^2(\omega)k_\rho^2}\begin{bmatrix} k_x^2 K_{z,i}(\omega) & k_x k_y K_{z,i}(\omega) & k_x k_\rho^2 \\ k_x k_y K_{z,i}(\omega) & k_y^2 K_{z,i}(\omega) & k_y k_\rho^2 \\ -k_x k_\rho^2 & -k_y k_\rho^2 & -k_\rho^4/K_{z,i}(\omega)\end{bmatrix}$$

$$R_{\text{s}} = \frac{K_{z,i}(\omega)-K_{z,j}(\omega)}{K_{z,i}(\omega)+K_{z,j}(\omega)} \qquad R_{\text{p}} = \frac{\epsilon_{\text{r},j}(\omega)K_{z,i}(\omega)-\epsilon_{\text{r},i}(\omega)K_{z,j}(\omega)}{\epsilon_{\text{r},j}(\omega)K_{z,i}(\omega)+\epsilon_{\text{r},i}(\omega)K_{z,j}(\omega)} \qquad K_{z,i}(\omega)=\sqrt{\epsilon_{\text{r},i}(\omega)\frac{\omega^2}{c^2}-k_x^2-k_y^2} \qquad k_\rho^2 = k_x^2+k_y^2$$

In cylindrical coordinates:

$$x-x' = \rho\cos(\phi) \qquad y-y' = \rho\sin(\phi) \qquad \phi = \tan^{-1}\left(\frac{y-y'}{x-x'}\right)$$

$$k_x = k_\rho\cos(\alpha) \qquad k_y = k_\rho\sin(\alpha) \qquad \alpha = \tan^{-1}(k_y/k_x)$$



$$\therefore\ \overline{\overline{\mathbf{G}}}^{(i)}_{\text{refl}}(\rho,\phi,z,z',\omega) = \frac{i}{8\pi^2}\int_0^{+\infty} dk_\rho \int_0^{2\pi} d\alpha\, k_\rho \left\{\overline{\overline{\mathbf{M}}}_{\text{s}}(k_\rho,\alpha,\omega)+\overline{\overline{\mathbf{M}}}_{\text{p}}(k_\rho,\alpha,\omega)\right\} e^{ik_\rho\rho\cos(\phi)\cos(\alpha)+ik_\rho\rho\sin(\phi)\sin(\alpha)}e^{iK_{z,i}(k_\rho,\omega)(z+z')}$$

where:

$$\overline{\overline{\mathbf{M}}}_{\text{s}}(k_\rho,\alpha,\omega) = \frac{R_{\text{s}}(k_\rho,\omega)}{K_{z,i}(k_\rho,\omega)k_\rho^2}\begin{bmatrix} k_\rho^2\sin^2(\alpha) & -k_\rho^2\sin(\alpha)\cos(\alpha) & 0 \\ -k_\rho^2\sin(\alpha)\cos(\alpha) & k_\rho^2\cos^2(\alpha) & 0 \\ 0 & 0 & 0\end{bmatrix}$$

$$\overline{\overline{\mathbf{M}}}_{\text{p}}(k_\rho,\alpha,\omega) = \frac{-R_{\text{p}}(k_\rho,\omega)}{k_i^2(\omega)k_\rho^2}\begin{bmatrix} k_\rho^2\cos^2(\alpha)K_{z,i}(k_\rho,\omega) & k_\rho^2\sin(\alpha)\cos(\alpha)K_{z,i}(k_\rho,\omega) & k_\rho\cos(\alpha)k_\rho^2 \\ k_\rho^2\sin(\alpha)\cos(\alpha)K_{z,i}(k_\rho,\omega) & k_\rho^2\sin^2(\alpha)K_{z,i}(k_\rho,\omega) & k_\rho\sin(\alpha)k_\rho^2 \\ -k_\rho\cos(\alpha)k_\rho^2 & -k_\rho\sin(\alpha)k_\rho^2 & -k_\rho^4/K_{z,i}(k_\rho,\omega)\end{bmatrix}$$

$$R_{\text{s}}(k_\rho,\omega) = R_{\text{s}}$$

$$R_{\text{p}}(k_\rho,\omega) = R_{\text{p}}$$

$$K_{z,i}(k_\rho,\omega) = K_{z,i}(\omega) = \sqrt{\epsilon_{\text{r},i}(\omega)\frac{\omega^2}{c^2}-k_\rho^2}$$

S-Wave in Cylindrical Coordinates

$$\overline{\overline{\mathbf{G}}}^{(i)}_{\text{refl}} = \overline{\overline{\mathbf{G}}}^{(i)}_{\text{refl,s}} + \overline{\overline{\mathbf{G}}}^{(i)}_{\text{refl,p}}$$

$$\overline{\overline{\mathbf{G}}}^{(i)}_{\text{refl,s}}(\rho,\phi,z,z',\omega) = \frac{i}{8\pi^2}\int_0^{+\infty} dk_\rho \int_0^{2\pi} d\alpha\, k_\rho \left\{\overline{\overline{\mathbf{M}}}_{\text{s}}(k_\rho,\alpha,\omega)\right\} e^{ik_\rho\rho\cos(\phi)\cos(\alpha)+ik_\rho\rho\sin(\phi)\sin(\alpha)} e^{iK_{z,i}(k_\rho,\omega)(z+z')}$$

$$= \frac{i}{8\pi^2}\int_0^{+\infty} dk_\rho \int_0^{2\pi} d\alpha\, k_\rho \left\{\overline{\overline{\mathbf{M}}}_{\text{s}}(k_\rho,\alpha,\omega)\right\} e^{ik_\rho\rho\cos(\alpha-\phi)} e^{iK_{z,i}(k_\rho,\omega)(z+z')}$$

where: $\overline{\overline{\mathbf{M}}}_{\text{s}}(k_\rho,\alpha,\omega) = \frac{R_{\text{s}}(k_\rho,\omega)}{K_{z,i}(k_\rho,\omega)}\begin{bmatrix} \sin^2(\alpha) & -\sin(\alpha)\cos(\alpha) & 0 \\ -\sin(\alpha)\cos(\alpha) & \cos^2(\alpha) & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Define a rotation matrix: $\overline{\overline{\mathbf{T}}} = \begin{bmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\overline{\overline{\mathbf{G}}}^{(i)}_{\text{refl,s}}(\rho,\phi,z,z',\omega) = \frac{i}{8\pi^2}\int_0^{+\infty} dk_\rho \int_0^{2\pi} d\alpha\, k_\rho \overline{\overline{\mathbf{T}}}\left\{\overline{\overline{\mathbf{T}}}^{-1}\overline{\overline{\mathbf{M}}}_{\text{s}}(k_\rho,\alpha,\omega)\overline{\overline{\mathbf{T}}}\right\}\overline{\overline{\mathbf{T}}}^{-1} e^{ik_\rho\rho\cos(\alpha-\phi)} e^{iK_{z,i}(k_\rho,\omega)(z+z')}$$

$$= \frac{i}{8\pi^2}\overline{\overline{\mathbf{T}}}\int_0^{+\infty} dk_\rho \int_0^{2\pi} d\alpha\, k_\rho \left\{\frac{R_{\text{s}}(k_\rho,\omega)}{2K_{z,i}(k_\rho,\omega)}\begin{bmatrix} 1-\cos(2(\alpha-\phi)) & -\sin(2(\alpha-\phi)) & 0 \\ -\sin(2(\alpha-\phi)) & 1+\cos(2(\alpha-\phi)) & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\} e^{ik_\rho\rho\cos(\alpha-\phi)} e^{iK_{z,i}(k_\rho,\omega)(z+z')}\overline{\overline{\mathbf{T}}}^{-1}$$

Use the integral representation of Bessel function: $J_n(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{i(x\sin(\theta)-n\theta)}d\theta$

We can derive:

$$J_0(k_\rho\rho) = \frac{1}{2\pi}\int_0^{2\pi} 1 e^{ik_\rho\rho\cos(\alpha-\phi)}d\alpha$$

$$J_2(k_\rho\rho) = \frac{-1}{2\pi}\int_0^{2\pi} \cos(2(\alpha-\phi))e^{ik_\rho\rho\cos(\alpha-\phi)}d\alpha$$

$$0 = \int_0^{2\pi} \sin(2(\alpha-\phi))e^{ik_\rho\rho\cos(\alpha-\phi)}d\alpha$$

$$\overline{\overline{\mathbf{G}}}^{(i)}_{\text{refl,s}} = \frac{i}{4\pi}\overline{\overline{\mathbf{T}}}\int_0^{+\infty} dk_\rho\, k_\rho \left\{\frac{R_{\text{s}}(k_\rho,\omega)}{2K_{z,i}(k_\rho,\omega)}\begin{bmatrix} J_0(k_\rho\rho)+J_2(k_\rho\rho) & 0 & 0 \\ 0 & J_0(k_\rho\rho)-J_2(k_\rho\rho) & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\} e^{iK_{z,i}(k_\rho,\omega)(z+z')}\overline{\overline{\mathbf{T}}}^{-1}$$

S-Wave in Cylindrical Coordinates

$$\overline{\overline{\mathbf{G}}}^{(i)}_{\text{refl,s}}(\rho, \phi, z, z', \omega) = \frac{i}{4\pi} \overline{\overline{\mathbf{T}}} \int_0^{+\infty} dk_\rho \, k_\rho \left\{ \frac{R_\text{s}(k_\rho, \omega)}{2K_{z,i}(k_\rho, \omega)} \begin{bmatrix} J_0(k_\rho \rho) + J_2(k_\rho \rho) & 0 & 0 \\ 0 & J_0(k_\rho \rho) - J_2(k_\rho \rho) & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\} e^{iK_{z,i}(k_\rho,\omega)(z+z')} \overline{\overline{\mathbf{T}}}^{-1}$$

$$= \int_0^{+\infty} \frac{idk_\rho}{4\pi} R_\text{s}(k_\rho, \omega) \left\{ \frac{k_\rho}{2K_{z,i}(k_\rho, \omega)} \begin{bmatrix} J_0(k_\rho \rho) + \cos(2\phi) J_2(k_\rho \rho) & \sin(2\phi) J_2(k_\rho \rho) & 0 \\ \sin(2\phi) J_2(k_\rho \rho) & J_0(k_\rho \rho) - \cos(2\phi) J_2(k_\rho \rho) & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\} e^{iK_{z,i}(k_\rho,\omega)(z+z')}$$

$\therefore$ $$\overline{\overline{\mathbf{G}}}^{(i)}_{\text{refl,s}}(\rho, \phi, z, z', \omega) = \int_0^{+\infty} \frac{idk_\rho}{4\pi} R_\text{s}(k_\rho, \omega) \overline{\overline{\mathbf{M}}}_\text{s}(k_\rho, \omega) e^{iK_{z,i}(k_\rho,\omega)(z+z')}$$

where: $$\overline{\overline{\mathbf{M}}}_\text{s}(k_\rho, \omega) = \frac{k_\rho}{2K_{z,i}(k_\rho, \omega)} \begin{bmatrix} J_0(k_\rho \rho) + \cos(2\phi) J_2(k_\rho \rho) & \sin(2\phi) J_2(k_\rho \rho) & 0 \\ \sin(2\phi) J_2(k_\rho \rho) & J_0(k_\rho \rho) - \cos(2\phi) J_2(k_\rho \rho) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_s(k_\rho, \omega) = \frac{K_{z,i}(k_\rho, \omega) - K_{z,j}(k_\rho, \omega)}{K_{z,i}(k_\rho, \omega) + K_{z,j}(k_\rho, \omega)}$$ and $$K_{z,i}(k_\rho, \omega) = \sqrt{\epsilon_{\text{r},i}(\omega) \frac{\omega^2}{c^2} - k_\rho^2}$$

P-Wave in Cylindrical Coordinates

$$\overline{\overline{\mathbf{G}}}^{(i)}_{\mathrm{refl}} = \overline{\overline{\mathbf{G}}}^{(i)}_{\mathrm{refl,s}} + \overline{\overline{\mathbf{G}}}^{(i)}_{\mathrm{refl,p}}$$

$$\overline{\overline{\mathbf{G}}}^{(i)}_{\mathrm{refl,p}}(\rho,\phi,z,z',\omega) = \frac{i}{8\pi^2}\int_0^{+\infty} dk_\rho \int_0^{2\pi} d\alpha\, k_\rho \left\{\overline{\overline{\mathbf{M}}}_{\mathrm{p}}(k_\rho,\alpha,\omega)\right\} e^{ik_\rho\rho\cos(\alpha-\phi)} e^{iK_{z,i}(k_\rho,\omega)(z+z')}$$

where: $$\overline{\overline{\mathbf{M}}}_{\mathrm{p}}(k_\rho,\alpha,\omega) = \frac{-R_{\mathrm{p}}(k_\rho,\omega)}{k_i^2(\omega)} \begin{bmatrix} \cos^2(\alpha)K_{z,i}(k_\rho,\omega) & \sin(\alpha)\cos(\alpha)K_{z,i}(k_\rho,\omega) & k_\rho\cos(\alpha) \\ \sin(\alpha)\cos(\alpha)K_{z,i}(k_\rho,\omega) & \sin^2(\alpha)K_{z,i}(k_\rho,\omega) & k_\rho\sin(\alpha) \\ -k_\rho\cos(\alpha) & -k_\rho\sin(\alpha) & -k_\rho^2/K_{z,i}(k_\rho,\omega) \end{bmatrix}$$

Use the same rotation matrix: $$\overline{\overline{\mathbf{T}}} = \begin{bmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\overline{\overline{\mathbf{G}}}^{(i)}_{\mathrm{refl,p}}(\rho,\phi,z,z',\omega) = \frac{i}{8\pi^2}\overline{\overline{\mathbf{T}}}\int_0^{+\infty} dk_\rho \int_0^{2\pi} d\alpha\, k_\rho \left\{\overline{\overline{\mathbf{T}}}^{-1}\overline{\overline{\mathbf{M}}}_{\mathrm{p}}(k_\rho,\alpha,\omega)\overline{\overline{\mathbf{T}}}\right\} e^{ik_\rho\rho\cos(\alpha-\phi)} e^{iK_{z,i}(k_\rho,\omega)(z+z')}\overline{\overline{\mathbf{T}}}^{-1}$$

$$= \frac{i}{8\pi^2}\overline{\overline{\mathbf{T}}}\int_0^{+\infty} dk_\rho \int_0^{2\pi} d\alpha\, k_\rho \left\{\frac{-R_{\mathrm{p}}(k_\rho,\omega)}{k_i^2(\omega)} \begin{bmatrix} \frac{1+\cos(2(\alpha-\phi))}{2}K_{z,i}(k_\rho,\omega) & \frac{\sin(2(\alpha-\phi))}{2}K_{z,i}(k_\rho,\omega) & k_\rho\cos(\alpha-\phi) \\ \frac{\sin(2(\alpha-\phi))}{2}K_{z,i}(k_\rho,\omega) & \frac{1-\cos(2(\alpha-\phi))}{2}K_{z,i}(k_\rho,\omega) & k_\rho\sin(\alpha-\phi) \\ -k_\rho\cos(\alpha-\phi) & -k_\rho\sin(\alpha-\phi) & -k_\rho^2/K_{z,i}(k_\rho,\omega) \end{bmatrix}\right\} e^{ik_\rho\rho\cos(\alpha-\phi)} e^{iK_{z,i}(k_\rho,\omega)(z+z')}\overline{\overline{\mathbf{T}}}^{-1}$$

Use the integral representation of Bessel function:

$$0 = \int_0^{2\pi} \sin(\alpha-\phi)e^{ik_\rho\rho\cos(\alpha-\phi)}d\alpha \qquad 0 = \int_0^{2\pi} \sin(2(\alpha-\phi))e^{ik_\rho\rho\cos(\alpha-\phi)}d\alpha \qquad J_0(k_\rho\rho) = \frac{1}{2\pi}\int_0^{2\pi} 1e^{ik_\rho\rho\cos(\alpha-\phi)}d\alpha$$

$$J_1(k_\rho\rho) = \frac{-i}{2\pi}\int_0^{2\pi} \cos(\alpha-\phi)e^{ik_\rho\rho\cos(\alpha-\phi)}d\alpha \qquad J_2(k_\rho\rho) = \frac{-1}{2\pi}\int_0^{2\pi} \cos(2(\alpha-\phi))e^{ik_\rho\rho\cos(\alpha-\phi)}d\alpha$$

P-Wave in Cylindrical Coordinates

$$\overline{\overline{\mathbf{G}}}^{(i)}_{\text{refl,p}}(\rho,\phi,z,z',\omega) = \frac{i}{8\pi^2}\overline{\overline{\mathbf{T}}}\int_0^{+\infty} dk_\rho \int_0^{2\pi} d\alpha\, k_\rho \left\{\overline{\overline{\mathbf{T}}}^{-1}\overline{\overline{\mathbf{M}}}_\text{p}(k_\rho,\alpha,\omega)\overline{\overline{\mathbf{T}}}\right\} e^{ik_\rho\rho\cos(\alpha-\phi)} e^{iK_{z,i}(k_\rho,\omega)(z+z')}\overline{\overline{\mathbf{T}}}^{-1}$$

$$= \frac{i}{4\pi}\overline{\overline{\mathbf{T}}}\int_0^{+\infty} dk_\rho\, k_\rho \left\{\frac{-R_\text{p}(k_\rho,\omega)}{k_i^2(\omega)}\begin{bmatrix} \frac{J_0(k_\rho\rho)-J_2(k_\rho\rho)}{2}K_{z,i}(k_\rho,\omega) & 0 & ik_\rho J_1(k_\rho\rho) \\ 0 & \frac{J_0(k_\rho\rho)+J_2(k_\rho\rho)}{2}K_{z,i}(k_\rho,\omega) & 0 \\ -ik_\rho J_1(k_\rho\rho) & 0 & -k_\rho^2 J_0(k_\rho\rho)/K_{z,i}(k_\rho,\omega) \end{bmatrix}\right\} e^{iK_{z,i}(k_\rho,\omega)(z+z')}\overline{\overline{\mathbf{T}}}^{-1}$$

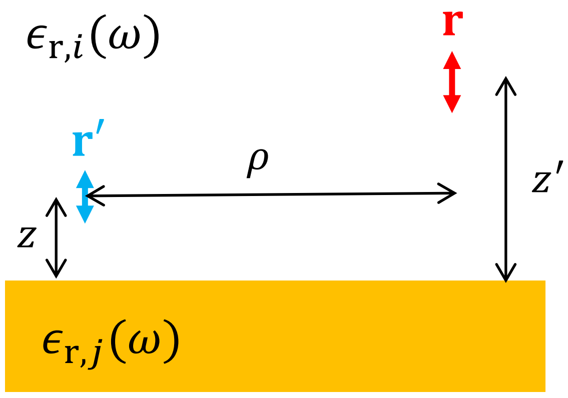
$$= \frac{i}{4\pi}\int_0^{+\infty} dk_\rho\, k_\rho \left\{\frac{-R_\text{p}(k_\rho,\omega)K_{z,i}(k_\rho,\omega)}{2k_i^2(\omega)}\begin{bmatrix} J_0(k_\rho\rho)-\cos(2\phi)J_2(k_\rho\rho) & -\sin(2\phi)J_2(k_\rho\rho) & \frac{2ik_\rho}{K_{z,i}(k_\rho,\omega)}\cos(\phi)J_1(k_\rho\rho) \\ -\sin(2\phi)J_2(k_\rho\rho) & J_0(k_\rho\rho)+\cos(2\phi)J_2(k_\rho\rho) & \frac{2ik_\rho}{K_{z,i}(k_\rho,\omega)}\sin(\phi)J_1(k_\rho\rho) \\ \frac{-2ik_\rho}{K_{z,i}(k_\rho,\omega)}\cos(\phi)J_1(k_\rho\rho)) & \frac{-2ik_\rho}{K_{z,i}(k_\rho,\omega)}\sin(\phi)J_1(k_\rho\rho) & \frac{-2k_\rho^2}{K_{z,i}^2(k_\rho,\omega)}J_0(k_\rho\rho) \end{bmatrix}\right\} e^{iK_{z,i}(k_\rho,\omega)(z+z')}$$

$$\therefore \qquad \overline{\overline{\mathbf{G}}}^{(i)}_{\text{refl,p}}(\rho,\phi,z,z',\omega) = \int_0^{+\infty}\frac{idk_\rho}{4\pi} R_\text{p}(k_\rho,\omega)\overline{\overline{\mathbf{M}}}_\text{p}(k_\rho,\omega)e^{iK_{z,i}(k_\rho,\omega)(z+z')}$$

where: $$\overline{\overline{\mathbf{M}}}_\text{p}(k_\rho,\omega) = \frac{-k_\rho K_{z,i}(k_\rho,\omega)}{2k_i^2(\omega)}\begin{bmatrix} J_0(k_\rho\rho)-\cos(2\phi)J_2(k_\rho\rho) & -\sin(2\phi)J_2(k_\rho\rho) & \frac{2ik_\rho}{K_{z,i}(k_\rho,\omega)}\cos(\phi)J_1(k_\rho\rho) \\ -\sin(2\phi)J_2(k_\rho\rho) & J_0(k_\rho\rho)+\cos(2\phi)J_2(k_\rho\rho) & \frac{2ik_\rho}{K_{z,i}(k_\rho,\omega)}\sin(\phi)J_1(k_\rho\rho) \\ \frac{-2ik_\rho}{K_{z,i}(k_\rho,\omega)}\cos(\phi)J_1(k_\rho\rho)) & \frac{-2ik_\rho}{K_{z,i}(k_\rho,\omega)}\sin(\phi)J_1(k_\rho\rho) & \frac{-2k_\rho^2}{K_{z,i}^2(k_\rho,\omega)}J_0(k_\rho\rho) \end{bmatrix}$$

$$R_\text{p}(k_\rho,\omega) = \frac{\epsilon_{\text{r},j}(\omega)K_{z,i}(k_\rho,\omega) - \epsilon_{\text{r},i}(\omega)K_{z,j}(k_\rho,\omega)}{\epsilon_{\text{r},j}(\omega)K_{z,i}(k_\rho,\omega) + \epsilon_{\text{r},i}(\omega)K_{z,j}(k_\rho,\omega)}$$ and $$K_{z,i}(k_\rho,\omega) = \sqrt{\epsilon_{\text{r},i}(\omega)\frac{\omega^2}{c^2} - k_\rho^2}$$

Summary



$$\overline{\overline{\mathbf{G}}}^{(i)}(\mathbf{r},\mathbf{r}',\omega) = \overline{\overline{\mathbf{G}}}_0^{(i)}(\mathbf{r},\mathbf{r}',\omega) + \overline{\overline{\mathbf{G}}}_{\text{refl}}^{(i)}(\mathbf{r},\mathbf{r}',\omega)$$

$$\overline{\overline{\mathbf{G}}}_0^{(i)}(\mathbf{r},\mathbf{r}',\omega) = \frac{e^{ik_i(\omega)R}}{4\pi k_i^2(\omega)R}\left\{\left(\overline{\overline{\mathbf{I}}} - \hat{\mathbf{R}}\hat{\mathbf{R}}\right)k_i^2(\omega) + \left(3\hat{\mathbf{R}}\hat{\mathbf{R}} - \overline{\overline{\mathbf{I}}}\right)\left(\frac{1}{R^2} - \frac{ik_i(\omega)}{R}\right)\right\}$$

$$\overline{\overline{\mathbf{G}}}_{\text{refl}}^{(i)}(\mathbf{r},\mathbf{r}',\omega) = \overline{\overline{\mathbf{G}}}_{\text{refl}}^{(i)}(\rho,\phi,z,z',\omega) = \int_0^{+\infty}\frac{idk_\rho}{4\pi}\left[R_{\text{s}}(k_\rho,\omega)\overline{\overline{\mathbf{M}}}_{\text{s}}(k_\rho,\omega) + R_{\text{p}}(k_\rho,\omega)\overline{\overline{\mathbf{M}}}_{\text{p}}(k_\rho,\omega)\right]e^{iK_{z,i}(k_\rho,\omega)(z+z')}$$

where:

$$\overline{\overline{\mathbf{M}}}_{\text{s}}(k_\rho,\omega) = \frac{k_\rho}{2K_{z,i}(k_\rho,\omega)}\begin{bmatrix} J_0(k_\rho\rho) + \cos(2\phi)J_2(k_\rho\rho) & \sin(2\phi)J_2(k_\rho\rho) & 0 \\ \sin(2\phi)J_2(k_\rho\rho) & J_0(k_\rho\rho) - \cos(2\phi)J_2(k_\rho\rho) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and

$$R_s(k_\rho,\omega) = \frac{K_{z,i}(k_\rho,\omega) - K_{z,j}(k_\rho,\omega)}{K_{z,i}(k_\rho,\omega) + K_{z,j}(k_\rho,\omega)}$$

$$\overline{\overline{\mathbf{M}}}_{\text{p}}(k_\rho,\omega) = \frac{-k_\rho K_{z,i}(k_\rho,\omega)}{2k_i^2(\omega)}\begin{bmatrix} J_0(k_\rho\rho) - \cos(2\phi)J_2(k_\rho\rho) & -\sin(2\phi)J_2(k_\rho\rho) & \frac{2ik_\rho}{K_{z,i}(k_\rho,\omega)}\cos(\phi)J_1(k_\rho\rho) \\ -\sin(2\phi)J_2(k_\rho\rho) & J_0(k_\rho\rho) + \cos(2\phi)J_2(k_\rho\rho) & \frac{2ik_\rho}{K_{z,i}(k_\rho,\omega)}\sin(\phi)J_1(k_\rho\rho) \\ \frac{-2ik_\rho}{K_{z,i}(k_\rho,\omega)}\cos(\phi)J_1(k_\rho\rho)) & \frac{-2ik_\rho}{K_{z,i}(k_\rho,\omega)}\sin(\phi)J_1(k_\rho\rho) & \frac{-2k_\rho^2}{K_{z,i}^2(k_\rho,\omega)}J_0(k_\rho\rho) \end{bmatrix}$$

and $R_p(k_\rho,\omega) = \frac{\epsilon_{r,j}(\omega)K_{z,i}(k_\rho,\omega) - \epsilon_{r,i}(\omega)K_{z,j}(k_\rho,\omega)}{\epsilon_{r,j}(\omega)K_{z,i}(k_\rho,\omega) + \epsilon_{r,i}(\omega)K_{z,j}(k_\rho,\omega)}$ and $K_{z,i}(k_\rho,\omega) = \sqrt{\epsilon_{r,i}(\omega)\frac{\omega^2}{c^2} - k_\rho^2}$ $\quad \phi = \tan^{-1}\left(\frac{y-y'}{x-x'}\right)$